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DIFFUSION AND MULTIVELOCITY MODELS OF TWO-PHASE MEDIA IN AN ELECTRIC FIELD*

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The motion of fluid containing gas bubbles or incompressible particles in an electric field is considered. A new model of disperse medium in which a separate equation of motion is devised for each phase, with the presence of other phases taken into account by adding to equations terms related to interphase momentum exchange, the "multivelocity" approximation, is devised. Formulas are obtained for forces generated by discrepancies in the permittivity and conductivity of phases, which are exerted by the electric field on each phase. Formulas for phenomenological coefficients in the equations of "diffusion" and "multivelocity" models are also obtained. Derivation of the mixture energy equation is presented for the case of a multivelocity uneven-temperature medium. The physical meaning of terms appearing in the equations is explained, and the structure of equations which define the electric field effect on bubble volume is discussed.

The new effect of adding to the equation of motion the force acting on the dispersed phase associated with the medium polarization is investigated. That force represents an additional "pressure gradient" which imparts to the mixture of two incompressible phases the properties and characteristics of a compressible medium. The medium compressibility manifests itself by the appearance of weak perturbations at a finite velocity induced by the electric field, in which the dispersed phase volume concentration changes together with the mixture permittivity, the electric and other parameters.

The propagation velocity of weak discontinuities is determined in the case of the multivelocity and diffusion models. It is shown that simplification or complication of the system of equations, as well as the addition to equations of terms that define the interaction between the electric field and the polarizable medium may lead to a change of the type of equations, which must be taken into consideration in the formulation and solution of specific problems.

The fundamentals of polarizable and magnetizable media hydrodynamics are set forth in the monograph /1/. Derivation of equations defining the motion of multicomponent and disperse systems appears in /2—4/ in the diffusion approximation in which each component or phase are polarized, generally in conformity with their specific laws.

1. The equations of motion of a fluid with gas bubbles in an electric field. The diffusion approximation.

Let us consider the motion of such fluid, assuming the fluid incompressible and the gas in bubbles perfect.

In the considered here problem of a two-phase medium on the assumption of equal temperatures of phases, of absence of chemical reactions and of free charges, of the possibility of neglecting the mixture viscosity, and that the mixture permittivity depends only on the volume concentration of bubbles, the equations of motion determined without the constraints appearing in /3,4/ are of the form (in the diffusion approximation cross effects are disregarded)

$$\frac{\partial\rho}{\partial t}+\operatorname{div}\rho\mathbf{u}=0,\quad \frac{d\Gamma}{dt}+(\Gamma-1)\operatorname{div}\mathbf{u}=-\frac{\operatorname{div}\mathbf{J}}{\rho_1^\circ},\quad \mathbf{J}=\rho_2(\mathbf{v}_2-\mathbf{u}) \tag{1.1}$$

$$\rho_1^\circ=\text{const},\quad \rho=\rho_1+\rho_2,\quad \rho_1=\rho_1^\circ(1-\Gamma),\quad \rho_2=\rho_2^\circ\Gamma$$

$$\rho\frac{du_i}{dt}=-\frac{\partial}{\partial x_i}p+\frac{\partial}{\partial x_k}\left\{\frac{E_iD_k}{4\pi}-\left(\varepsilon-\Gamma\frac{\partial\varepsilon}{\partial\Gamma}\right)\frac{E^2}{8\pi}\delta_{ik}\right\}+\rho g_i,\qquad \rho\mathbf{u}=\rho_1\mathbf{v}_1+\rho_2\mathbf{v}_2 \tag{1.2}$$

$$\mathbf{J}=TL\left\{\nabla(\xi_{01}-\xi_{02})+\frac{1}{\rho_2^\circ}\nabla p_{02}+\left(\frac{1}{\rho_1^\circ}-\frac{1}{\rho_2^\circ}\right)\nabla p+\frac{1}{\rho_2^\circ}\nabla\frac{\partial\varepsilon}{\partial\Gamma}\frac{E^2}{8\pi}\right\} \tag{1.3}$$

$$\frac{\partial}{\partial t}\left(\rho\frac{u^2}{2}+\rho U\right)=-\frac{\partial}{\partial x_i}\left\{\rho u_i\left(\frac{u^2}{2}+U\right)+u_i\left[p-\frac{E^2}{8\pi}\left(\varepsilon+\Gamma\frac{\partial\varepsilon}{\partial\Gamma}\right)\right]+\frac{c}{4\pi}[\mathbf{EH}]_i-\varkappa\nabla_iT-J_il\right\} \tag{1.4}$$

$$U=\sum_1^2 c_\alpha U_{0\alpha}+\varepsilon(\Gamma)\frac{E^2}{8\pi\rho},\quad U_{0\alpha}=c_{v\alpha}T$$

$$\rho_2\frac{d_2}{dt}\frac{\Gamma}{\rho_2}=L_\Gamma\left(p_{02}+\frac{\partial\varepsilon}{\partial\Gamma}\frac{E^2}{8\pi}-p\right),\quad p_{02}=\rho_2^\circ R_2T \tag{1.5}$$

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$$\xi_{01} = U_{01} - s_{01}T = f_1(T), \quad \xi_{02} = U_{02} - s_{02}T + \frac{p_{02}}{\rho_2^\circ} = f_2(T, \rho_2^\circ), \quad s_{01} = c_{v1}\ln T, \quad s_{02} = c_{v2}\ln T - R_2 \ln \rho_2^\circ \tag{1.6}$$

$$l = \xi_{01} - \xi_{02} + \frac{p_{02}}{\rho_2^\circ} + \left(\frac{1}{\rho_1^\circ} - \frac{1}{\rho_2^\circ}\right) p + \frac{1}{\rho_2^\circ}\frac{\partial \varepsilon}{\partial \Gamma}\frac{E^2}{8\pi}$$
$$\operatorname{rot} \mathbf{E} = 0, \quad \operatorname{div} \mathbf{D} = 0, \quad \mathbf{D} = \varepsilon(\Gamma)\mathbf{E} \tag{1.7}$$

where $\rho_\alpha^\circ, c_\alpha, \mathbf{v}_\alpha, U_{0\alpha}$, and $s_{0\alpha}$ are the true density, mass concentration, velocity, internal energy, and the entropy of a unit mass, respectively (in the absence of field) of the α phase with $\alpha = 1$ for the fluid and $\alpha = 2$ for the gas; Γ is the volume concentration of bubbles; $\mathbf{u}$ and T are, respectively, the mixture mean mass velocity and temperature; p is the pressure in the carrier medium denoted in /3/ by $p^{inc}/(1-\Gamma)$, c_{v2} and c_{p2} are the specific heats of gas at constant volume and pressure, respectively; $R_2 = c_{p2} - c_{v2}$ is the gas constant; c_{v1} and $\varkappa$ are, respectively, the fluid specific heat and the thermal conductivity coefficient; $\mathbf{g}$ is the free fall acceleration vector; $\mathbf{E}$, $\mathbf{D}$, and ε are, respectively, the electric field intensity, electric induction and the mixture permittivity; L and L_Γ are coefficients of kinetic equations denoted in /3/ by $T^{-2}L_{2,2}$ and $\varphi_{10,10}T$, respectively.

The mixture permittivity is dependent only on the portion of the volume taken by the dispersed phase, which occurs, for instance, in the case when the permittivities of the fluid and bubbles can be assumed constant. When the permittivity depends on the medium temperature, the electric field, and on other parameters, the equation derived in /2—4/ differ from (1.2) —(1.7).

Equations (1.1)—(1.7) were derived using general principles of continuous medium mechanics and thermodynamics of irreversible processes.

Discontinuities of electric characteristics at the bubble—fluid interface is simulated by the dependence of mixture permittivity ε on the volume concentration Γ and on constants that define the electric properties of both media. This is related to the appearance in the right-hand side of the equation of motion (1.2) of striction type terms $\partial[\Gamma(\partial\varepsilon/\partial\Gamma)E^2/8\pi]/\partial x_i$ besides the derivatives of the Maxwellian stress tensor $\partial(E_iD_k/4\pi - \varepsilon E^2\delta_{ik}/8\pi)/\partial x_k$. The sum of these two terms is $\Gamma\partial[(\partial\varepsilon/\partial\Gamma)E^2/8\pi]/\partial x_i$ and cannot, generally speaking, be represented in the form of a gradient of some scalar quantity.

The last term in braces in the diffusion equation (1.3) defines the motion of bubbles (particles) relative to the fluid in an inhomogeneous electric field. The force that generates such motion is also related to the presence of a discontinuity of electric properties of media at the bubble—fluid interface.

The presence in equations of motion and diffusion of forces related to polarization of the medium as a whole may also occur when it is possible to disregard the polarization of each of the phases, i.e. when the phase permittivities ε_1 and ε_2 are equal unity. Effective polarization of the mixture may, in that case, be induced by, for instance, difference in the conductivities of phases and other factors.

Equation (1.5) is used for determining the volume concentration of bubbles, and links the pressures in bubbles and in fluid, and with the variation of bubble volume, taking into account the presence of the electric field and of the discontinuity of electric properties of the medium at the bubble— fluid interface.

In the considered case the medium as a whole is compressible $\operatorname{div}\mathbf{u} \neq 0$. However, when the carrier phase is incompressible, pressure p is determined, as in an incompressible medium, by the solution of system (1.1)—(1.7) with the condition $\rho_1^\circ = \text{const}$.

2. The connection between equations of motion in the diffusion and multivelocity models. The system of equations derived in Sect.1 consists of equations of motion and energy for the mixture as a whole, and of diffusion relationships which convert them into equations of motion and energy for each phase. In a number of cases this system does not define sufficiently fully the phenomena occurring in such media. This is, for example, related to the neglect in the diffusion equation of inertia terms, etc.

The equations of continuity and momentum for each phase (the multivelocity model) are of the form

$$\frac{\partial}{\partial t}(1-\Gamma) + \operatorname{div}(1-\Gamma)\mathbf{v}_1 = 0, \quad \rho_1 = \rho_1^\circ(1-\Gamma), \quad \rho_1^\circ = \text{const}, \qquad \frac{\partial}{\partial t}\rho_2^\circ\Gamma + \operatorname{div}\rho_2^\circ\Gamma\mathbf{v}_2 = 0 \tag{2.1}$$

$$\rho_1\frac{d_1v_{1i}}{dt} = -(1-\Gamma)\nabla_i p + f_{1,2}^i + F_{1i}^E + \rho_1^\circ(1-\Gamma)g_i + \nabla_k\Pi^{ik} \tag{2.2}$$
$$\rho_2\frac{d_2v_{2i}}{dt} = -\Gamma\nabla_i p - f_{1,2}^i + F_{2i}^E + \rho_2^\circ\Gamma g_i, \quad \mathbf{f}_{1,2} = L_f(\mathbf{v}_2 - \mathbf{v}_1)$$

where $T = \text{const}$ is assumed for definiteness.

The first of Eqs. (2.2) is the equation of motion of the carrier phase, and the second the equation of motion of the dispersed phase; $\mathbf{f}_{1,2}$ represents the momentum exchange, i.e. the "friction force" between phases: $\mathbf{F}_1^E$ and $\mathbf{F}_2^E$ are the forces exerted by the electric field on the carrier and dispersed phases. Adding Eqs. (2.2) we obtain

$$\rho \frac{du_i}{dt} + \nabla_k v_{12}^{ik} = -\nabla_i p + F_{1i}^E + F_{2i}^E + \rho g_i + \nabla_k \Pi^{ik}, \qquad v_{12}^{ik} = \frac{\rho_1\rho_2}{\rho}(v_1^i - v_2^i)(v_1^k - v_2^k) \tag{2.3}$$

We can introduce, as in the hydrodynamics of multicomponent media /5/, parameters p^* and Π_{ik}^*, using formula

$$-p^*\delta_{ik} + \Pi_{ik}^* = -p\delta_{ik} + \Pi_{ik} - v_{12}^{ik}$$

Parameters p, p^* and Π_{ik}, Π_{ik}^* represent the pressure and the viscous stress tensor in systems of coordinates moving, respectively, at the carrier medium and mixture velocities. A detailed exposition of the physical meaning of p, p^* and Π_{ik}, Π_{ik}^* appeared in /5/.

We subtract the second of Eqs. (2.2) divided by density ρ_2 from the first divided by density ρ_1 and obtain

$$\frac{d_1 v_{1i}}{dt} - \frac{d_2 v_{2i}}{dt} = \left(\frac{1}{\rho_2^\circ} - \frac{1}{\rho_1^\circ}\right)\nabla_i p + f_{12}^i \frac{\rho}{\rho_1\rho_2} + \frac{F_{1i}^E}{\rho_1} - \frac{F_{2i}^E}{\rho_2} + \frac{1}{\rho_1}\nabla_k \Pi^{ik} \tag{2.4}$$

Neglecting in the left-hand side of Eq. (2.3) the second term and the viscous forces $\nabla_k\Pi^{ik}$ and in Eq. (2.4) the viscous forces $\nabla_k\Pi^{ik}/\rho_1$ and the convection terms in its left-hand side, but retaining the terms proportional to the interphase friction force (respective estimates of these are readily obtained) and allowing for the equality $\mathbf{f}_{12} = L_f(\mathbf{v}_2 - \mathbf{v}_1) = L_f\rho(\mathbf{v}_2 - \mathbf{u})/\rho_1$, we obtain

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \mathbf{F}_1^E + \mathbf{F}_2^E + \rho \mathbf{g}, \qquad \mathbf{J} = \rho_2(\mathbf{v}_2 - \mathbf{u}) = \frac{\rho_1^2\rho_2^2}{\rho^2 L_f}\left\{\left(\frac{1}{\rho_1^\circ} - \frac{1}{\rho_2^\circ}\right)\nabla p + \frac{\mathbf{F}_2^E}{\rho_2} - \frac{\mathbf{F}_1^E}{\rho_1}\right\} \tag{2.5}$$

To compare the models of the diffusion and multivelocity approximations we write the equations of motion (1.2) for the mixture and the equation of diffusion (1.3) in the isothermic case

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \Gamma\nabla \frac{\partial\varepsilon}{\partial\Gamma}\frac{E^2}{8\pi} + \rho \mathbf{g}, \qquad \mathbf{J} = LT\left\{\left(\frac{1}{\rho_1^\circ} - \frac{1}{\rho_2^\circ}\right)\nabla p + \frac{1}{\rho_2^\circ}\nabla\frac{\partial\varepsilon}{\partial\Gamma}\frac{E^2}{8\pi}\right\} \tag{2.6}$$

Comparison of Eqs. (2.5) and (2.6) yields expressions for $\mathbf{F}_1^E$ and $\mathbf{F}_2^E$, and the relation between coefficients L and L_f

$$\mathbf{F}_1^E = 0, \quad \mathbf{F}_2^E = \Gamma\nabla\frac{\partial\varepsilon}{\partial\Gamma}\frac{E^2}{8\pi}, \quad L = \frac{\rho_1^2\rho_2^2}{\rho^2 T}L_f^{-1} \tag{2.7}$$

To find the expressions for coefficients L_f and L we assume that the particles or bubbles are identical, and that the friction force between fluid and dispersed particles per unit of the medium volume is equal to the sum of friction forces between the fluid and each particle in a unit volume, as defined by the Stokes formula, with the coefficient $L_f = 6\pi\mu_1 an$ (μ_1 is the dynamic viscosity of the carrier fluid, a is the particle or bubble radius, and n is the number of particles or bubbles in a unit volume of the mixture). In this case the formula for the coefficient L is

$$L = \frac{\rho_1^2\rho_2^2}{6\pi\mu_1 an\rho^2 T} \tag{2.8}$$

Note that the Stokes formula for the friction force acting on a bubble is only valid if the bubble behaves as if it had a solid boundary, which is possible, for instance, in the presence of surface-active substances or charges spread over its surface. Formula (2.8) in the case of low volume concentration of bubbles $\Gamma \ll 1$ and low real density of bubbles $\rho_2^\circ \ll \rho_1^\circ$ is of the form

$$L = T^{-1}\frac{(\rho_2^\circ)^{4/3}\Gamma m}{6\pi\mu_1(3m/4\pi)^{1/3}} \tag{2.9}$$

where m is the mass of a bubble.

Formula (2.7) implies that the electric field force related to the polarization of the medium as a whole acts only on the dispersed phase $\mathbf{F}_1^E = 0$. This is the consequence of the particular form of the dissipation function used in /2—4/ in the construction of the model of polarizable multiphase media. Note that in the approximation of fluid incompressibility, pressure p in the equation of motion for both phases is determined by the solution of the problem, and may contain terms related to the medium polarization. Such terms may also appear in expressions for the viscous stress tensor of the carrier phase in terms of chemical potentials /3,4/.

Assuming that the force $\mathbf{F}_2^E = n\mathbf{f}^E$ exerted by the electric field on bubbles is the sum of identical forces $\mathbf{f}^E$ acting on a single bubble of radius a, and using the second of formulas (2.7), we can write for force $\mathbf{f}^E$ the formula

$$\mathbf{f}^E = \frac{4}{3}\pi a^3\nabla\frac{\partial\varepsilon}{\partial\Gamma}\frac{E^2}{8\pi} \tag{2.10}$$

When the gas filling the bubbles and the carrier fluid are nonconducting, the permittivities of fluid ε_1 and of gas ε_2 may be assumed constant and $\Gamma \ll 1$, and the following formula holds:

$$\partial\varepsilon/\partial\Gamma = 3\varepsilon_1(\varepsilon_2 - \varepsilon_1)/(2\varepsilon_1 + \varepsilon_2), \quad \varepsilon = \varepsilon_1 + \Gamma\partial\varepsilon/\partial\Gamma \tag{2.11}$$

while the formula for the force $\mathbf{f}^E$ acting on a single bubble is of the form

$$\mathbf{f}^E = \frac{1}{2} a^3 \frac{\varepsilon_1 (\varepsilon_2 - \varepsilon_1)}{\varepsilon_2 + 2\varepsilon_1} \nabla E^2$$

which is the same as the formula proposed in /6/.

3. The formula for bubble volume variation. The usually used Rayleigh equation for defining the motion of a fluid with gas bubbles cannot be applied in the case of polarizable media, and is here replaced by Eq. (1.5). To understand the phenomena occurring in the considered here medium it is necessary to clarify the physical meaning of terms appearing in Eq. (1.5).

3.1. In the absence of evaporation and condensation the velocity $v_f(a)$ at the bubble boundary is equal to the rate of variation of the spherical bubble radius $a^{\cdot}$. In the case of identical bubbles

$$v_f(a) = a^{\cdot} = \frac{\rho_2}{4\pi a^2 n} \frac{d_2(\Gamma/\rho_2)}{dt} \tag{3.1}$$

When the electric field is absent, the condition of continuity of the momentum flux at the bubble-fluid interface, can be written, neglecting the viscosity of gas /1/ and taking into account formula (3.1), as

$$p_g - p_f = \frac{4\mu_1}{a} v_f(a) = \frac{4\mu_1\rho_2}{3\Gamma} \frac{d_2(\Gamma/\rho_2)}{dt} \tag{3.2}$$

where p_f and p_g are the fluid and gas pressures, respectivley, near the fluid-gas interface.

When the bubble is fairly small, its pressure can be assumed uniform and equal p_g; if the radial motion of fluid surrounding the bubble is neglected, the pressure of the fluid around a single bubble is also uniform and equal p_f. When the number of bubbles in the mixture is fairly small $\Gamma \ll 1$, the effect of one bubble on another, either directly or indirectly through effect on the carrier fluid, can be neglected. It is then possible to assume that the pressure in the gas and in the fluid is, respectively $p_{02} = p_g$ and $p = p_f$, and use for defining their relation the equation

$$\frac{3\Gamma}{4\mu_1}(p_{02} - p) = \rho_2 \frac{d_2(\Gamma/\rho_2)}{dt} \tag{3.3}$$

The comparison of Eq. (3.3) with (1.5), proposed here, yields for the coefficent L_Γ, in the absence of an electric field, the following formula:

$$L_\Gamma = 3\Gamma / (4\mu_1) \tag{3.4}$$

It follows from the above that the kinetic equation (1.5) for the variation of bubble volume, derived in /2—4/, may be treated as the condition at the discontinuity surface for some mean pressure of fluid and gas, taking into account the viscous stress tensor in the fluid. The term $L_\Gamma^{-1}\rho_2 d_2(\Gamma/\rho_2)/dt$ in Eq. (1.5) is proportional to the fluid velocity at the bubble boundary and takes into account the carrier phase viscosity.

Note that formula (3.4) was derived on the assumption of constant pressure in bubbles and smallness of their volume concentration, neglecting the viscosity of gas and the fluid kinetic energy associated with bubble pulsation. If even one of these conditions is violated, formula (3.4) may no longer be valid.

3.2. The term $(\partial\varepsilon/\partial\Gamma)E^2/(8\pi)$ in Eq. (1.5) defines the electric field effect on the bubble volume variation. When $\partial\varepsilon/\partial\Gamma$ is determined by formulas (2.11), this term is equal to the remainder of momentum fluxes in the fluid and gas, associated with the electric field and averaged over the surface of a bubble. Thus, Eq. (1.5) has the meaning of the averaged condition of continuity of momentum flux at the bubble—fluid interface, even in the presence of the electric field.

To prove this we consider a spherical bubble of nonconducting gas with permittivity ε_2 in a nonconducting fluid with permittivity ε_1 in the field $\mathbf{E}_\infty$ uniform at infinity. Let us show that the difference of projections of the tensor of Maxwellian stresses T_{ij} outside and inside the bubble on the normal to its surface is

$$\langle\langle\{T_{ij}n^i n^j\}\rangle\rangle = \frac{3\varepsilon_1(\varepsilon_2 - \varepsilon_1)}{\varepsilon_2 + 2\varepsilon_1} \cdot \frac{E_\infty^2}{8\pi} = \frac{\partial\varepsilon}{\partial\Gamma} \frac{E_\infty^2}{8\pi}, \quad T_{ij} = \frac{E_i^M D_j^M}{4\pi} - \frac{\mathbf{E}^M \mathbf{D}^M}{8\pi} \delta_{ij}$$

where E_i^M and D_j^M are components of the true electric field and induction.

The difference of projections of the Maxwellian stress tensor on the normal to the gas-fluid interface is

$$\{T_{ij}n^i n^j\} = \frac{(D_n^M)^2}{8\pi}\left\{\frac{1}{\varepsilon}\right\} - \frac{(E_\tau^M)^2}{8\pi}\{\varepsilon\} \tag{3.5}$$

It is assumed here that the electric induction normal component D_n^M and the electric field tangential component E_τ^M are continuous along the bubble boundary, and the notation $\{A\} = A_1 - A_2$ is used; A_1 and A_2 are the values of A in the fluid and gas, respectively.

The electric field $\mathbf{E}_{int}^M$ inside the gas bubble in the dielectric fluid and in field $\mathbf{E}_\infty$

uniform at infinity, is homogeneous and determined by the formula

$$\mathbf{E}^M_{\text{int}} = 3\varepsilon_1 \mathbf{E}_\infty / (2\varepsilon_1 + \varepsilon_2) \tag{3.6}$$

The quantities $D_n{}^M$ and $E_\tau{}^M$ at the bubble surface are, with allowance for (3.6), of the form

$$D_n{}^M = \frac{3\varepsilon_1\varepsilon_2}{2\varepsilon_1 + \varepsilon_2} E_\infty \sin\theta, \quad E_\tau{}^M = \frac{3\varepsilon_1}{2\varepsilon_1 + \varepsilon_2} E_\infty \cos\theta \tag{3.7}$$

where θ and α are spherical coordinates of a point on the bubble surface, and the z-axis is directed along vector $\mathbf{E}_\infty$.

Taking into account (3.7) and averaging formula (3.5) over the bubble surface, we obtain

$$\frac{1}{4\pi a^2} \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} a^2 \{T_{ij} n^i n^j\} \cos\theta \, d\alpha \, d\theta = \frac{3\varepsilon_1(\varepsilon_2 - \varepsilon_1) E^2_\infty}{8\pi(2\varepsilon_1 + \varepsilon_2)} = \frac{\partial\varepsilon}{\partial\Gamma} \frac{E^2_\infty}{8\pi} = \frac{\partial\varepsilon}{\partial\Gamma} \frac{E^2}{8\pi} \tag{3.8}$$

The last equality in formula (3.8) is valid if the field E_∞ at considerable distance from particles can be assumed equal to the field E defined by the Maxwell equation (1.7).

4. The equations of energy in the multivelocity model. Using the equations of continuity and motion (2.1) and (2.2), the equation of energy for the electromagnetic field in the case of nonconducting medium (i.e. the electrohydrodynamics approximation)

$$\frac{\mathbf{E}}{4\pi} \cdot \frac{\partial \mathbf{D}}{\partial t} = -\operatorname{div}\left\{\frac{c}{4\pi}[\mathbf{EH}]\right\} \tag{4.1}$$

and, the Gibbs identities for the internal energy of each phase, in the absence of field $U_{0\alpha}$ ($s_{0\alpha}$ is the entropy of the α-phase in the absence of electric field)

$$dU_{01} = T_1\, ds_{01}, \quad dU_{02} = T_2\, ds_{02} + \frac{p_{02}}{(\rho_2{}^\circ)^2} d\rho_2{}^\circ \tag{4.2}$$

and taking into account formulas (2.7) for the force $\mathbf{F}_1{}^E, \mathbf{F}_2{}^E$, in conformity with /2–4/, we obtain

$$\begin{aligned} \frac{\partial}{\partial t}\left(\rho_1 U_{01} + \rho_2 U_{02} + \rho_1 \frac{v_1^2}{2} + \rho_2 \frac{v_2^2}{2} + \varepsilon(\Gamma)\frac{E^2}{8\pi}\right) = -\operatorname{div}\left\{\rho_1 \mathbf{v}_1\left(U_{01} + \frac{v_1^2}{2} + \frac{p}{\rho_1{}^\circ}\right) + \rho_2 \mathbf{v}_2\left(U_{02} + \frac{v_2^2}{2} + \frac{p}{\rho_2{}^\circ}\right) - \right. \\ \left. \Gamma \mathbf{v}_2 \frac{\partial\varepsilon}{\partial\Gamma} \cdot \frac{E^2}{8\pi} + \frac{c}{4\pi}[\mathbf{EH}] + \mathbf{q}\right\} + T_1\left[\frac{\partial}{\partial t}(\rho_1 s_{01} + \rho_2 s_{02}) + \right. \\ \left. \operatorname{div}(\rho_1 s_{01}\mathbf{v}_1 + \rho_2 s_{02}\mathbf{v}_2)\right] + \rho_2 \frac{d_2 s_{02}}{dt}(T_2 - T_1) + \mathbf{f}_{12}(\mathbf{v}_1 - \mathbf{v}_2) + \rho_2 \frac{d_2(\Gamma/\rho_2)}{dt}\left(p - p_{02} - \frac{\partial\varepsilon}{\partial\Gamma}\frac{E^2}{8\pi}\right) + \operatorname{div}\mathbf{q} = 0 \end{aligned} \tag{4.3}$$

Note that equality (4.3) was derived on the assumption that the mixture permittivity depends only on the volume concentration Γ.

Let us assume that the equation of energy for the medium and field is of the form

$$\begin{aligned} \frac{\partial}{\partial t}\left(\rho_1 U_{01} + \rho_2 U_{02} + \rho_1 \frac{v_1^2}{2} + \rho_2 \frac{v_2^2}{2} + \varepsilon(\Gamma)\frac{E^2}{8\pi}\right) = \\ -\operatorname{div}\left\{\rho_1 \mathbf{v}_1\left(U_{01} + \frac{v_1^2}{2} + \frac{p}{\rho_1{}^\circ}\right) + \rho_2 \mathbf{v}_2\left(U_{02} + \frac{v_2^2}{2} + \frac{p}{\rho_2{}^\circ}\right) - \Gamma\frac{\partial\varepsilon}{\partial\Gamma}\frac{E^2}{8\pi}\mathbf{v}_2 + \frac{c}{4\pi}[\mathbf{EH}] + \mathbf{q}\right\} \end{aligned} \tag{4.4}$$

We define the mixture entropy by the formula $\rho S = \rho_1 s_{01} + \rho_2 s_{02}$. When $\varepsilon = \varepsilon(\Gamma)$ the mixture entropy S is equal its entropy outside the field.

The equation for the variation of entropy S is of the form

$$\begin{aligned} \frac{\partial}{\partial t}\rho S + \operatorname{div}\left(\rho_1 s_{01}\mathbf{v}_1 + \rho_2 s_{02}\mathbf{v}_2 + \frac{\mathbf{q}}{T_1}\right) = \frac{\rho_2}{T_1}\frac{d_2 s_{02}}{dt}(T_1 - T_2) + \\ \mathbf{f}_{12} T_1{}^{-1}(\mathbf{v}_2 - \mathbf{v}_1) - \frac{\rho_2}{T_1}\frac{d_2(\Gamma/\rho_2)}{dt}\left(p - p_{02} - \frac{\partial\varepsilon}{\partial\Gamma}\frac{E^2}{8\pi}\right) - \mathbf{q}\frac{\nabla T_1}{T_1^2} \end{aligned} \tag{4.5}$$

We assume that reversible entropy flux is $\rho_1 s_{01}\mathbf{v}_1 + \rho_2 s_{02}\mathbf{v}_2 + \mathbf{q}/T_1$ and the dissipative function σ of the form

$$\sigma = \rho_2 \frac{d_2 s_{02}}{dt}\frac{T_1 - T_2}{T_1} + \frac{\mathbf{f}_{12}}{T_1}(\mathbf{v}_2 - \mathbf{v}_1) - \frac{\rho_2}{T_1}\frac{d_2(\Gamma/\rho_2)}{dt}\left(p - p_{02} - \frac{\partial\varepsilon}{\partial\Gamma}\frac{E^2}{8\pi}\right) - \mathbf{q}\frac{\nabla T_1}{T_1^2} \tag{4.6}$$

Using the Onsager principle it is possible to obtain the kinetic equations which close the system of equations of the multivelocity model

$$\frac{\rho_2}{T_1}\frac{d_2 s_{02}}{dt} = \varphi_1(T_1 - T_2), \quad \rho_2\frac{d_2(\Gamma/\rho_2)}{dt} = L_\Gamma\left(p_{02} + \frac{\partial\varepsilon}{\partial\Gamma}\frac{E^2}{8\pi} - p\right), \quad \mathbf{f}_{12} = L_f(\mathbf{v}_2 - \mathbf{v}_1), \quad \mathbf{q} = -\varkappa\nabla T_1 \tag{4.7}$$

where, for simplicity, the cross effects have been omitted.

5. Properties of the system of equations which define polarizable disperse mixtures in various approximations. In formulating various problems it is important to know the propagation velocities of weak discontinuities and the type of applied equations. A simplification or complication of the system of equations may, generally speaking, result in a change of the equation type. Below, we consider the isothermal case of a multivelocity model and various approximations.

1°. The isothermic case of the multivelocity model of a mixture of fluid with gas bubbles when the dependence of ε on Γ $(\partial\varepsilon / \partial\Gamma = \text{const})$ is linear. The determining equations in dimensionless form are

$$\partial\Gamma / \partial t^* - (1-\Gamma)\nabla^* \mathbf{v}_1^* + \mathbf{v}_1^* \nabla\Gamma = 0 \tag{5.1}$$

$$\Gamma \partial\rho_2^{\circ *} / \partial t^* + \rho_2^{\circ *} \partial\Gamma / \partial t^* + \Gamma \mathbf{v}_2^* \nabla\rho_2^{\circ *} + \mathbf{v}_2^* \rho_2^{\circ *} \nabla\Gamma + \rho_2^{\circ *} \Gamma \nabla^* \mathbf{v}_2^* = 0 \tag{5.2}$$

$$\rho_1^{\circ *}(1-\Gamma)\partial \mathbf{v}_1^* / \partial t^* + \rho_1^{\circ *}(1-\Gamma)(\mathbf{v}_1^* \nabla^*)\mathbf{v}_1^* = -(1-\Gamma)\nabla^* p^* - St^{-1}\Gamma(\rho_2^{\circ *})^{2/3}(\mathbf{v}_1^* - \mathbf{v}_2^*) + (1-\Gamma)\rho_1^{\circ *} Fr^{-2}\mathbf{g}/g \tag{5.3}$$

$$\rho_2^{\circ *}\Gamma \partial \mathbf{v}_2^* / \partial t^* + \rho_2^{\circ *}\Gamma(\mathbf{v}_2^* \nabla^*)\mathbf{v}_2^* = -\Gamma\nabla^* p^* + St^{-1}\Gamma(\rho_2^{\circ *})^{2/3} \times (\mathbf{v}_1^* - \mathbf{v}_2^*) + \Gamma\rho_2^{\circ *} Fr^{-2}\mathbf{g}/g + (\partial\varepsilon^* / \partial\Gamma)\Gamma\nabla^* E^{*2} \tag{5.4}$$

$$\partial\Gamma / \partial t^* + \nabla^*\Gamma \mathbf{v}_2^* = -L_\Gamma (p^* - R^*\rho_2^{\circ *} - (\partial\varepsilon^* / \partial\Gamma)E^{*2}) \tag{5.5}$$

$$\text{rot}^* \mathbf{E}^* = 0, \quad \nabla^* \mathbf{D}^* = 0, \quad \mathbf{D}^* = \varepsilon^* \mathbf{E}^* \tag{5.6}$$

$$\rho_1^{\circ *} = \rho_1^\circ / \rho_0, \quad \rho_2^{\circ *} = \rho_2^\circ / \rho_0, \quad \mathbf{v}_1^* = \mathbf{v}_1 / v_0, \quad \mathbf{v}_2^* = \mathbf{v}_2 / v_0, \quad p^* = p / (\rho_0 v_0^2) \tag{5.7}$$

$$E^{*2} = \varepsilon_1 E^2 / (8\pi\rho_0 v_0^2), \quad R^* = R_2 T / v_0^2, \quad \varepsilon^* = \varepsilon / \varepsilon_1$$

$$\text{rot}^* = \text{rot}\cdot h, \quad \nabla^* = \nabla\cdot h, \quad t^* = t v_0 / h, \quad L_\Gamma^* = L_\Gamma \rho_0 v_0 h, \quad St^{-1} = \frac{6\pi\mu_1}{m^{2/3}}\left(\frac{3}{4\pi}\right)^{1/3}\frac{h}{\rho_0^{1/3} v_0}, \quad Fr^{-2} = gh / v_0^2$$

where ρ_0 and v_0 are characteristic values of the true density and velocity of particles, and h is a characteristic dimension of the problem. The system of Eqs. (5.1)—(5.7) is obtained from system (2.1), (2.2), (1.5), and (1.7) using formulas (2.7) and (2.9).

The equations for a mixture of fluid and solid particles on the same assumptions consist of equations of continuity (5.1) and (5.2), equations of motion (5.3) and (5.4) in which it is necessary to set $\rho_2^{\circ *} = 1$ $(\rho_0 = \rho_2^\circ = \text{const})$, and of Maxwell equations (5.6); it is necessary to set in formulas (5.7) $\rho_0 = \rho_2^\circ$.

In the one-dimensional unsteady case the propagation velocity λ of weak discontinuities in a medium defined by Eqs. (5.1)—(5.7) is

$$\lambda_1 = v_2, \quad \lambda_{2,3} = (v_1\Gamma\rho_1^\circ + v_2(1-\Gamma)\rho_2^\circ \pm \Delta^{1/2}) / [\rho_1^\circ\Gamma + \rho_2^\circ(1-\Gamma)] \tag{5.8}$$

$$\Delta = (1-\Gamma)\Gamma\{[\rho_1^\circ\Gamma + \rho_2^\circ(1-\Gamma)]2D^2(\partial\varepsilon / \partial\Gamma)^2\varepsilon^{-3} - \rho_1^\circ\rho_2^\circ(v_1 - v_2)^2\}, \quad D = \text{const}$$

(here and subsequently the asterisks are omitted and all parameters are dimensionless).

It is obvious that in the absence of an electric field $D = 0$, $\Delta < 0$, and there is only one propagation velocity $\lambda_1 = v_2$ of weak discontinuities. In the presence of an electric field and $\Delta \geqslant 0$ there may be two $(\Delta = 0)$ or three $(\Delta > 0)$ velocities.

2°. Let us consider the system of equations that define the motion of bubbles, when it is possible to disregard the acceleration of fluid—Eqs.(5.2)—(5.6) in which we set $d_1 v_1 / dt = 0$. The propagation velocities of weak discontinuities are

$$\lambda_1 = v_2, \quad \lambda_{2,3} = v_2 \pm [2D^2(\partial\varepsilon / \partial\Gamma)^2\Gamma / (\rho_2^\circ\varepsilon^3)]^{1/2} \tag{5.9}$$

In this case, unlike in formulas (5.8), $\lambda_{1,2,3}$ are always real. When the field is present there are three characteristics, in its absence only one.

3°. Let us consider the system of Eqs. (5.1)—(5.7) in which instead of the equations of motion (5.3) and (5.4) we use the equation of motion of the mixture, i.e. the sum of Eqs. (5.3) and (5.4) and the remainder of Eqs. (5.3) and (5.4) divided by $\rho_1^\circ(1-\Gamma)$ and $\rho_2^\circ\Gamma$, respectively. We disregard the remainder of convection terms, which are small in comparison with the friction force, in the equation obtained by the subtraction of Eqs. (5.4) and (5.3).

Note that the obtained simplified system differs from that of Eqs. (1.1)—(1.7) of the diffusion approximation by the equation of motion (1.2) for the mixture which in Case 3° contains the additional term $\nabla_k c_{12}^{ik}$. When the difference in velocities is small, for instance, in comparison with the mean mass velocity, this term may be negelected, and Case 3° is transformed in the diffusion approximation.

In Case 3° the formulas for the propagation velocity of weak discontinuities are of the form

$$\lambda_1 = v_2, \quad \lambda_{2,3} = (v_2\rho_2^\circ - v_1\rho_1^\circ \pm \Delta_c^{1/2}) / (\rho_2^\circ - \rho_1^\circ), \quad \Delta_c = \rho_1^\circ\rho_2^\circ(v_1 - v_2)^2 + 2D^2\rho(\partial\varepsilon / \partial\Gamma)^2 / \varepsilon^3 \tag{5.10}$$

Formulas (5.10) imply that in Case 3° of the simplified system of equations, unlike in the multivelocity model, there are always three characteristic velocities, even in the absence of field.

4°. Let us consider the system of Eqs. (1.1)—(1.7), which defines the motion of fluid with gas bubbles in the diffusion approximation which is a further simplification of system

(5.1) — (5.7) in conformity with the exposition in Sects.1 and 2. Propagation velocities of weak discontinuities are determined by formulas

$$\lambda_1 = v_2, \quad \lambda_{2,3} = u \pm [2D^2 (\partial\varepsilon / \partial\Gamma)^2 \rho / \varepsilon^3]^{1/2} / (\rho_2^\circ - \rho_1^\circ) \tag{5.11}$$

The propagation velocities of weak discontinuities of equations defining the motion of fluid with incompressible particles are determined in Cases $1^\circ - 4^\circ$ by the second relations of (5.8) — (5.11), respectively, with $\rho_2^\circ \equiv 1$. The velocities determined by the first equalities of formulas (5.8) — (5.11) are absent (*).

The presence of an electric field considerably affects the propagation velocity of weak discontinuities in an incompressible fluid. In Cases 1°, 2°, and 4° not only the velocity changes but, also, the number of propagation velocities of weak discontinuities, i.e. the type of equations. New discontinuity propagation velocities may develop as the result of phase polarization in the electric field. There exist weak perturbations whose velocities coincide with weak discontinuity velocities.

Let us explain the physical meaning of the electric field effect on the propagation velocity of weak discontinuities in an incompressible fluid with incompressible particles whose permittivity differs from that of the fluid. We shall use Case 2° of the multivelocity model which defines the motion of bubbles when the effect of fluid acceleration on their flow can be neglected.

In the absence of an electric field, weak discontinuities travel at the dispersed phase velocity v_2. These are perturbations of the entropy wave type in which particle concentration Γ changes, while the remaining parameters are unaffected.

In the presence of an electric field a local perturbation of Γ results in the appearance of a local electric field gradient ∇E^2 and of a force proportional to ∇E^2 which acts on particles whose permittivity differs from that of the fluid. This force induces motion of particles, which diminishes the perturbation of the volume concentration Γ, similarly to the effect of pressure perturbation in a compressible gas, which induces the motion of gas molecules with the resulting decrease of density perturbations. The propagation of acoustic waves in compressible media is related to this effect. In a polarizable disperse medium consisting of incompressible phases the part of pressure gradient is played by the electric field gradient generated by perturbations of the dispersed phase volume concentration, as defined by Eq. .6).

Thus, the concentration perturbations in the presence of an electric field propagate at a velocity determined by the velocity of particles and the electric field intensity. The volume concentration of particles, the velocity, and the electric field change in these waves.

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*) The ellipticity of equations defining the motion of incompressible fluid with solid particles in the absence of electric field was pointed out in the report "Certain problems of fluidized bed hydrodynamics" by V. A. Polianskii, et.al. Report No.1691, Inst. Mekhaniki, MGU, 1975.